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Convection in vertically vibrated granular materials

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This paper briefly reviews some recent experiments in which we were able to measure the flow patterns of granular materials within a vertically vibrated container. The measurement methods include the use both of tracer beads to measure the vertical velocity along the centre of the container, and magnetic-resonance imaging to study the velocity profile in the radial direction. Most of the work concerns containers with vertical walls, although some results are mentioned that deal with convection in containers with walls that were splayed outwards.

Keywords: magnetic-resonance imaging; size separation; boundary effects; vibration; granular flow; velocity profile

In an 1831 paper to the Royal Society, Faraday first reported that granular material in a vertically vibrated container can spontaneously start to flow so that grains circulate from the top of the container to the bottom and then return back to the top surface (Faraday 1831). This motion resembles the convection rolls often seen in liquids heated from below. In vessels with vertical walls, the flow is upwards in the centre and downwards along the sides (Knight *et al*. 1993; Pak & Behringer 1993, 1994; Fauve *et al*. 1989; Evesque & Rajchenbach 1989; Ehrichs *et al*. 1995). When the side walls are splayed outwards, the convection direction can be reversed, so that the flow is downward in the centre of the container (Knight *et al*. 1993; Takahashi *et al*. 1968; Jaeger *et al*. 1994; Knight 1997).

In recent years, this phenomenon has been studied extensively by many groups interested in elucidating the cause of this behaviour (a brief review of this work can be found in Jaeger *et al*. (1996)). Two mechanisms appear to be particularly important for convection in rigid containers. In both cases, the maximum acceleration, A, during a vibration cycle must be greater than that of gravity, g. (In normalized units $\Gamma \equiv A/g > 1$.) The first mechanism is the one originally proposed by Faraday (1831), and involves the presence of interstitial gas which may entrain the grains in its flow. Experiments studying the role of gas originally gave conflicting results (Fauve *et al*. 1989; Evesque & Rajchenbach 1989). More recent experiments by Pak *et al*. (1995) have helped resolve the debate by carefully studying the dependence of the convection on the interstitial gas pressure. These results indicate that for very small grains, the

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presence of an interstitial gas, even at low pressure, may be relevant in causing a convective instability and heaps along the top surface of the pile.

The convection rolls can also be driven by friction with the container walls (Knight *et al*. 1993; Ehrichs *et al*. 1995; Taguchi 1992*a*, *b*; Thompson 1993; Luding *et al*. 1994*a*, *b*; Poeschel & Herrmann 1995). During each vibration cycle, the container walls produce a different amount of drag on the particles when they are moving upwards, with respect to the walls, than when they are falling. This leads to a net flow of the particles during each 'tap' (i.e. each full vibration cycle).

The cause of this asymmetric frictional force is the different density (and pressure) that the particles have when they are in flight during a shake, than when they are resting on the container bottom. When the particles are on the container bottom, their density is large, and the normal force on the side walls is large as well. This leads to a large friction when the particles start to rise with respect to the walls. Thus, as the particles leave the bottom during the vibration, the ones near the side wall are prevented from moving as freely as those in the middle of the container. However, when the particles are in free flight, their density is decreased and they no longer produce such a large normal pressure on the side walls. As they start to fall with respect to the container, the frictional forces with the side walls are less than they were when the beads were travelling upward. Thus, there is a net downward frictional force per cycle that forces the beads near the wall to travel towards the container bottom, leading to circulating flow. Even when there is no interstitial gas, these frictional effects with the side walls can drive the circulating convection flows.

One consequence of this convective motion is that it can lead to size separation in granular materials (Knight *et al*. 1993). Larger particles imbedded in a container filled with smaller ones, are brought to the top of a vibrated container of beads and then stranded there because they cannot fit into the thin region of downward-moving particles. This type of size separation is often an important problem in technological processes involving mixing of granular materials.

In recent experiments, we have measured the velocity profiles of the convection rolls in vertically vibrated containers (Ehrichs *et al*. 1995; Knight *et al*. 1996). These experiments employed both tracer-bead techniques and the technique of magnetic-resonance imaging (MRI), which was originally developed by Nakagawa *et al*. (1993) for studying flow in granular materials. MRI techniques are particularly well adapted to studying granular motion non-invasively deep inside the material, away from any walls. Since it is difficult to see inside a granular material optically, such techniques afford a unique view of granular dynamics. From such measurements, both the depth-dependence of the convection velocity and the detailed shape of the velocity profiles have been obtained (Ehrichs *et al*. 1995; Knight *et al*. 1996). The experiments show that the fastest flow occurs in the thin boundary layer near the walls. This differs from what occurs for a conventional fluid, where no-slip boundary conditions apply. This result, therefore, raises a number of issues about what are the correct boundary conditions to use for granular convection and other granular flows.

The tracer-bead techniques are particularly good at measuring the average upward flow in a situation where the convection rolls are very slowly moving. A single distinguishable bead is initially imbedded at a measured depth inside the container filled with other particles. We then measure the time (or the number of discrete taps) for that bead to reach the top surface, where it then becomes visible, as a function of its initial depth, and the normalized acceleration, Γ, and frequency, f, of the container

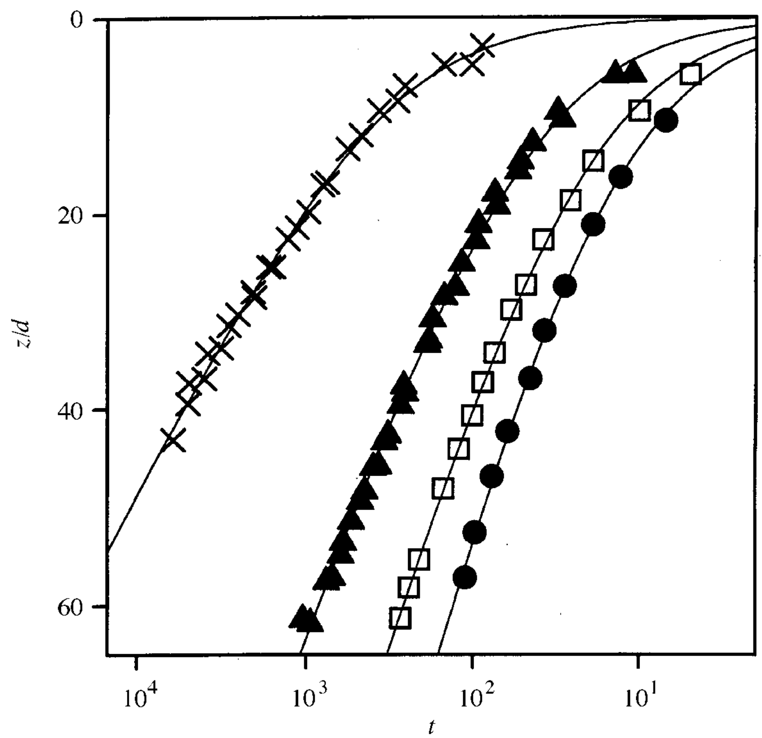

Figure 1. The initial depth, z/d (where z is the depth below the top surface and normalized to units of bead diameter, d), of a tracer particle, is plotted against the logarithm of the time (in units of 'taps') it takes for it to reach the top surface. The tracer particle was placed along the axis of a vertically vibrated cylinder filled with granular material. Each curve is for a different acceleration: $\Gamma = 3.0$ (×), $\Gamma = 6.0$ (▲), $\Gamma = 9.0$ (□) and $\Gamma = 12.0$ (•). The lines are fits of equation (1.1) to the data. Figure taken from Knight *et al.* (1996).

vibrations. Our studies indicate (Knight *et al.* 1996) that, along the container axis ($r = 0$), a particle rises to the surface from an initial depth z logarithmically in time, t:

$$z(r = 0) = \xi \ln[1 + t/\tau], \tag{1.1}$$

where ξ and τ are, respectively, a length- and a time-scale (for exceptions to this behaviour, see Knight *et al.* (1996)). Figure 1 shows behaviour of this sort for one set of vibration parameters. This implies that the particles rise in the container at a net average velocity, $v = -\mathrm{d}z/\mathrm{d}t$, which decays exponentially with the depth from the top surface:

$$v(r = 0) = -(\xi/\tau)\exp(-z/\xi). \tag{1.2}$$

Both ξ and τ depend on Γ and f, in addition to particle size and other physical parameters. We find that $\xi \propto 1/f^2$ when the acceleration is held constant, and $\xi \propto \Gamma$ when the frequency is kept fixed. Since $\Gamma = \delta(2\pi f)^2/g$, where δ is the amplitude of the taps, we can combine these results to find that ξ depends linearly on δ: $\xi \propto \Gamma/f^2 \propto \delta$. We also find that the time-scale, τ, of the convective flow increases dramatically as Γ decreases, and that τ appears to be exponential in frequency when the acceleration is held fixed: $\tau \propto \exp[f/f_0]$. A typical value for f_0 is 35 Hz, which is

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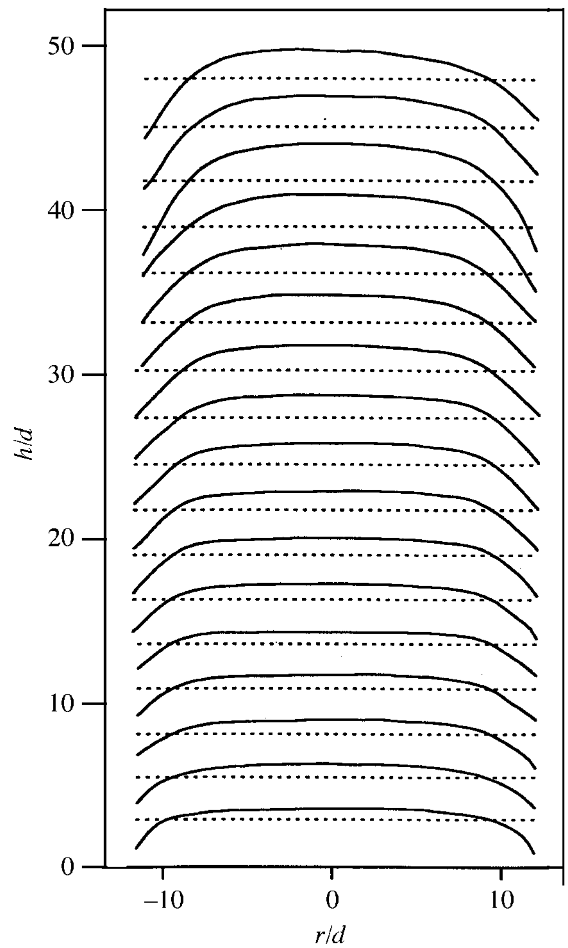

Figure 2. Each pair of solid and dotted lines shows the net motion of particles at normalized height, h/d, plotted against normalized radius, r/d, for particles in a vertically vibrated cylinder with $\Gamma = 6.0$. For each pair of curves, the motion is determined by measuring the difference between the final positions of the particles, given by the solid line, and their initial positions, given by the horizontal dotted line. The velocity is downwards at the container edge and upwards in the centre. Along the walls, the downward velocity is actually greater than the upward velocity in the centre. Figure taken from Knight *et al.* (1996).

approximately the frequency obtained by calculating the time it takes a particle to fall a distance equivalent to its own diameter, d, in gravity, g: $f_d \approx (g/d)^{0.5} \approx 50$ Hz.

In order to measure the radial variation of the convection velocity, we used MRI (Ehrichs *et al*. 1995; Knight *et al*. 1996). In this experiment, a cylinder was filled with poppy seeds and placed within the bore of an MRI magnet. We used poppy seeds because a good magnetic-resonance signal could be obtained from the free protons in the liquid oil contained within the seeds. The seeds were *ca.* 1 mm long and 0.75 mm wide. A monolayer of seeds was glued along the walls of the cylinder to provide a well-defined friction (Knight *et al*. 1993), as well as to provide a control layer from which to measure motion, as originally done in the work of Nakagawa *et al*. (1993).

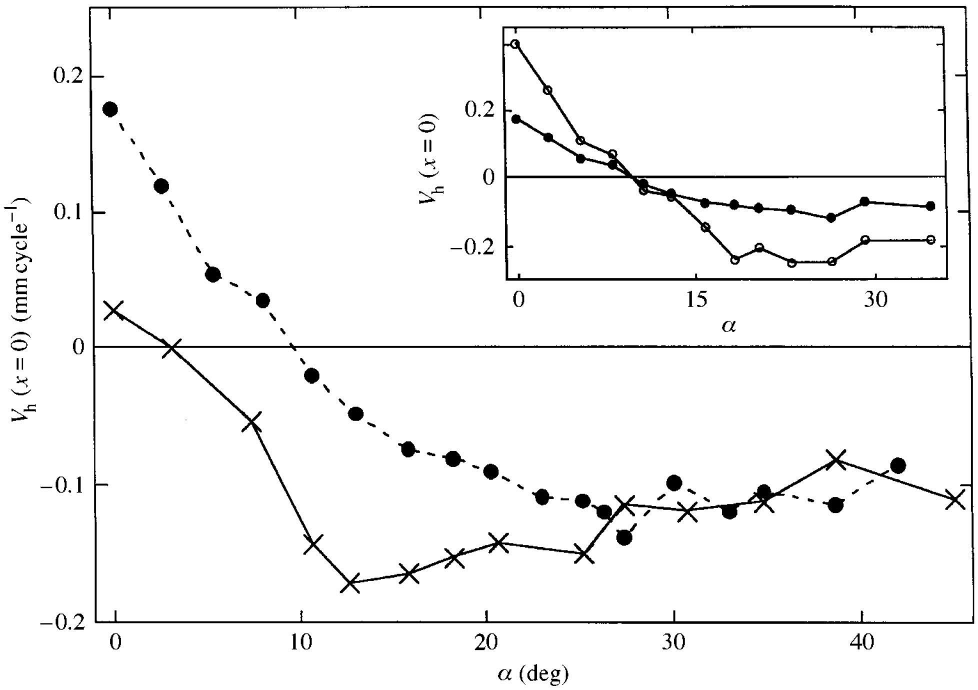

Figure 3. $v(x=0)$, the net average velocity in the centre of a container (vertically vibrated with $\Gamma = 4.2$ and $f = 25$ Hz), is plotted versus the wall angle, α. When the side walls are smooth ($\times$) the transition to a reversed flow occurs at a small angle, α, and when the walls are rough ($\bullet$) it occurs at a larger angle. The lines simply connect the data points. Inset shows the same measurement with rough walls for two different accelerations: $\Gamma = 4.2$ ($\bullet$) and $\Gamma = 5.6$ ($\circ$). Figure taken from Knight (1997).

Seeds were selectively 'tagged' by modulating the longitudinal spin polarization in the vertical direction. After the tagging, the container was vibrated one cycle and then the positions of the tagged particles were read. By determining the shift in position of the tagged particles, we were able to obtain the net average motion of the particles, both as a function of depth from the top surface and as a function of radius from the central axis of the cylinder. Figure 2 shows how the particles have moved during one tap throughout the cylinder. The velocity of the particle motion can be well fitted by an expression which has an exponential dependence on depth, as well as an exponential dependence on distance from the walls:

$$v(r,z) = -(\xi/\tau)[1 + B\{1 - \cosh(r/r_0)\}]\exp[-z/\xi], \qquad (1.3)$$

where B is a constant that depends on the radius of the container. In this fitting function, $\cosh(r/r_0)$ can be replaced by $I_0(r/r_0)$, the modified Bessel function of order zero, without having any discernible effect on the quality of the agreement.

We finally return to what we mentioned at the outset: the direction of the convection rolls can be reversed if the lateral walls are splayed outwards. In the container with vertical walls, the velocity of the particles is upwards at the centre and downwards in a thin stream along the walls. If the walls are slanted outwards, the velocity of the rolls initially starts to decrease, and after the walls have been slanted past a critical angle the direction of the rolls becomes reversed, so that the motion in the centre of the container is downwards (Knight 1997). Figure 3 shows the average velocity of the central particles as a function of angle for two different amounts of side-wall friction. In the case with the greater side-wall friction, the angle where the roll reversal occurs is larger. The inset of figure 3 shows that the critical angle where the roll reversal occurs is independent of the acceleration of the container.

An explanation for this effect is that, as the walls become more splayed from the vertical, the friction with the walls becomes less important during the upward motion of the particles (since they are moving with a component of velocity that is pointing away from the walls, rather than merely moving tangentially along the walls). Thus, the mechanism for the convection rolls in the vertical containers that we outlined above is no longer dominant. However, as the walls become more angled outward, there is more room for the particles to expand during their flight so that they hit the walls on their downward trajectory at distances farther and farther above their starting point. This leads to more friction (pointing upward along the walls) on their downward path as the particles slide back to the bottom of the container. This produces a convection roll which is now upward at the walls and downward in the centre, that is, reversed from the direction in the containers with vertical walls. It is clear from this explanation that the angle where roll reversal occurs will depend on the amount of friction of the beads with the side walls.

Convection, as we mentioned, can lead to size separation. Because size separation is such an important problem in industrial processes, it thus becomes important to understand convection in granular materials. Perhaps more important, however, is that convective motion in these materials provides a particularly good venue in which to study granular flow. Because the flow is continuously recirculating, transient effects can be minimized so that flow can be analysed in the steady state. Thus, the experimental data on the convection profiles provide an excellent benchmark for testing of both analytic theories and simulations that attempt to describe the flow of these materials. The techniques for visualizing the flow, such as the MRI methods described above and reported by Nakagawa (1993), are, we hope, a powerful tool that will prove widely applicable for future non-invasive experiments on these systems.

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